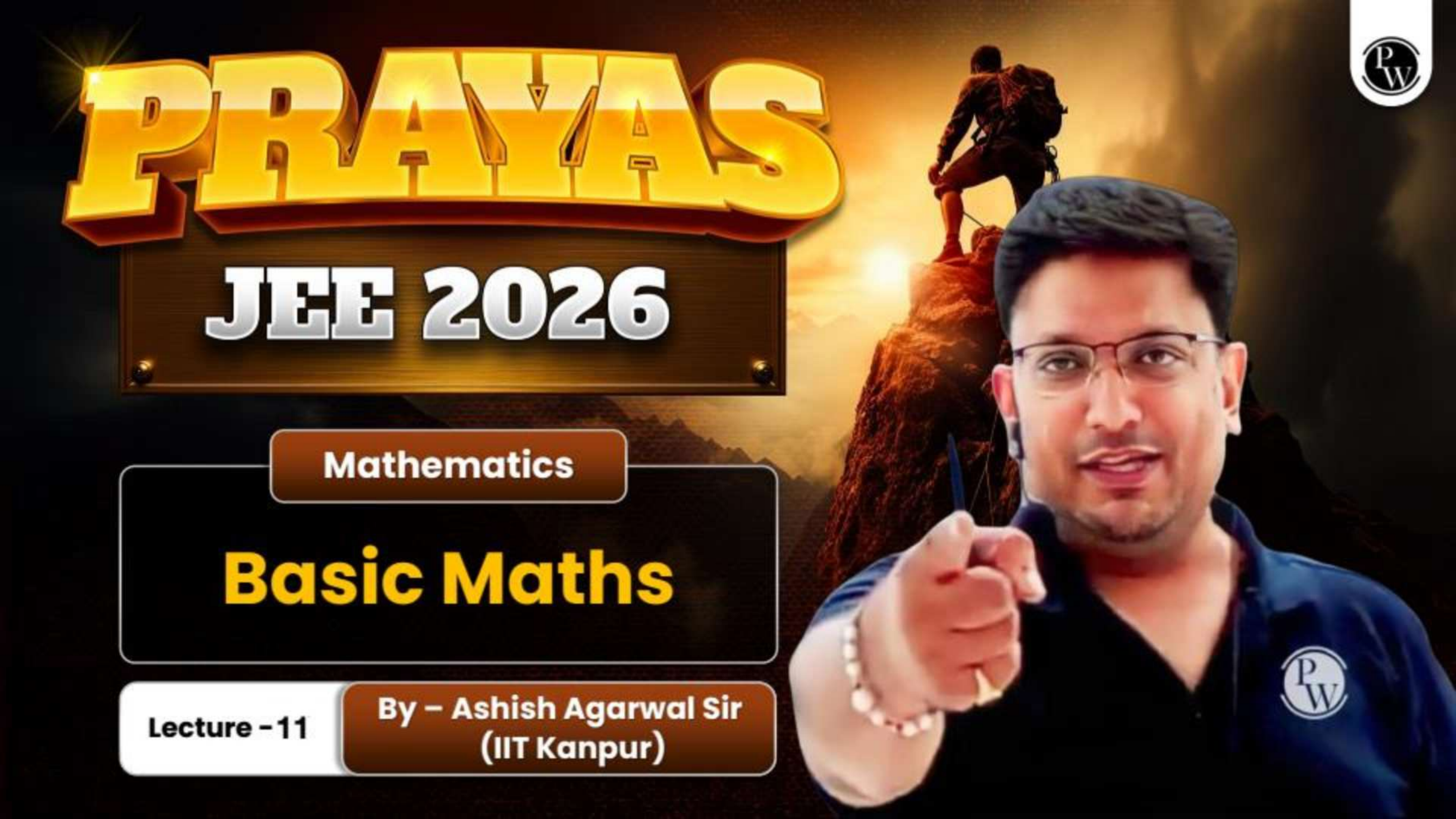
PRAYAS
JEE 2026
Mathematics
Basic Maths
Lecture -11
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PW

Topics *to be covered*

- **A** Logarithm Equations involving Modulus
- **B** Logarithmic Inequalities
- **C** Problem Practice

Homework Discussion

QUESTION

Indicate all correct alternatives, where base of the log is 2.

The equation $x^{\frac{3}{4}(\log_2 x)^2+\log_2 x-\frac{5}{4}} = \sqrt{2}$ has :

(A) At least one real solution

(B) Exactly 3 real solutions

(C) Exactly one irrational solution

(D) Imaginary roots

$\log_2 x^{\frac{3}{4}(\log_2 x)^2+\log_2 x-\frac{5}{4}} = \log_2 \sqrt{2}$

$\left(\frac{3}{4}(\log_2 x)^2+\log_2 x-\frac{5}{4}\right)\cdot \log_2 x = \frac{1}{2}$

let $\log_2 x = t$

$\frac{3}{4}t^3 + t^2 - \frac{5}{4}t = \frac{1}{2}$

$3t^3 + 4t^2 - 5t - 2 = 0$

$3t^2(t-1) + 7t(t-1) + 2(t-1) = 0$

$(t-1)(3t^2 + 6t + t + 2) = 0$

$(t-1)(3t+1)(t+2) = 0$

$t = 1, -\frac{1}{3}, -2$

$\log_2 x = 1, -\frac{1}{3}, -2$

$x = 2, 2^{-\frac{1}{3}}, 2^{-2}$

$x = 2, \frac{1}{4}, \frac{1}{\sqrt[3]{2}}$.

Irrational root

QUESTION

The equation $x^{\left[(\log_3 x)^2 - \frac{9}{2}\log_3 x + 5\right]} = 3\sqrt{3}$ has

Every Real no: is a complex Number

(A) Exactly 3 real solutions

$\log_3\left(x^{(\log_3 x)^2 - \frac{9}{2}\log_3 x + 5}\right) = \log_3 3\sqrt{3}$

(B) At least one real solution

$\left((\log_3 x)^2 - \frac{9}{2}\log_3 x + 5\right)\cdot \log_3 x = \frac{3}{2}$

(C) Exactly one irrational solution

$t^3 - \frac{9}{2}t^2 + 5t = \frac{3}{2}$

$2t^3 - 9t^2 + 10t - 3 = 0$

(D) Complex roots

$2t^2(t-1) - 7t(t-1) + 3(t-1) = 0$

$x = 3, \sqrt{3}, 27$

$(t-1)(2t^2 - 6t - t + 3) = 0$

$t = 1, \ (2t-1)(t-3) = 0$

$t = 1/2, 3$

QUESTION

(KTK 4)

Which of the following does not hold true for the expression $E = \sqrt{x^2 - 2x + 1} - \sqrt{x^2 + 2x + 1}$?

$E = |x-1| - |x+1|$

(A) $E = 2$ if $x \leq -1$

if $x \leq -1$

$E = -(x-1) + (x+1) = 2$

(B) $E = -2x$ if $-1 < x < 1$

if $-1 < x < 1$

$E = -(x-1) - (x+1) = -2x$

(C) $E = -2$ if $x \geq 1$

if $x \geq 1$

$E = x - 1 - (x+1) = -2$

(D) $E = -2$ for all x

Ans. D

Home Challenge-03

Positive integers a and b satisfy the condition $\log_2[\underbrace{\log_{2^a}(\log_{2^b}(2^{1000}))}_{t}] = 0$. Then the possible values of a + b is/are:

(A) ~~501~~

(B) ~~252~~

(C) ~~128~~

(D) 66

$\log_2 t = 0$

$t = 2^0 = 1$

$\Rightarrow \log_{2^a}(\log_{2^b} 2^{1000}) = 1$

$\frac{1}{a}\log_2\left(\frac{1000}{b}\log_2 2\right) = 1$

$\log_2\left(\frac{1000}{b}\right) = a$

$\frac{1000}{b} = 2^a$

$2^a \cdot b = 1000.$

$1000 = 2^1 \cdot 500$

$= 2^2 \cdot 250$

$= 2^3 \cdot 125.$

$\begin{cases} a=1, & b=500 \\ a=2, & b=250 \\ a=3, & b=125. \end{cases}$

Aao Machaay Dhamaal

Deh Swaal pe Deh Swaal

If a certain term
occurs repeatedly in
a given Eqn/Expression
assume it to be t

QUESTION

★★KCLS★★

Let $y = \sqrt{\log_2 3 \cdot \log_2 12 \cdot \log_2 48 \cdot \log_2 192 + 16} - \log_2 12 \cdot \log_2 48 + 10$. Find $y \in N$.

$$y = \sqrt{\log_2 3 \left(\log_2 4 + \log_2 3\right)\left(\log_2 16 + \log_2 3\right)\left(\log_2 64 + \log_2 3\right) + 16} - \left(\log_2 4 + \log_2 3\right)\left(\log_2 16 + \log_2 3\right) + 10$$

$$= \sqrt{\log_2 3\,(2 + \log_2 3)(4 + \log_2 3)(6 + \log_2 3) + 16} - (2 + \log_2 3)\cdot(4 + \log_2 3) + 10$$

let $\log_2 3 = t$

$$y = \sqrt{t(2+t)(4+t)(6+t) + 16} - (2+t)(4+t) + 10$$

$$y = \sqrt{(6t+t^2)(6t+t^2+8)+16} - (6t+t^2+8) + 10$$ let $m = t^2 + 6t$

$$= \sqrt{m(m+8)+16} - (m+8) + 10$$

$$= \sqrt{m^2+8m+16} - m + 2$$

$$y=\sqrt{(m+4)^2}-m+2$$

$$y=|m+4|-m+2$$

$+ve$

$$y=m+4-m+2=6.$$

$m=t^2+6t$, $t=\log_2 3>0$

$m>0$

QUESTION

$3^{\log_3^2 x} + x^{\log_3 x} = 162$ then x is

(A) 9

(B) 1/9

(C) 10

(D) 1/10

$\because \log_3 3^4 = 4\log_3 3 = 4$

$3^{\log_3^2 x} + x^{\log_3 x} = 162$

$\left(3^{\log_3 x}\right)^{\log_3 x} + x^{\log_3 x} = 162$

$x^{\log_3 x} + x^{\log_3 x} = 162$

$\cancel{2}\cdot x^{\log_3 x} = \cancel{162}$

$x^{\log_3 x} = 81$

$\log_3 x^{\log_3 x} = \log_3 81 \rightarrow \log_3 x \cdot \log_3 x = 4$

$(\log_3 x)^2 = 4 \Rightarrow \log_3 x = -2, 2$

$x = 3^{-2}, 3^{2}$

$x = \frac{1}{9}, 9$

QUESTION

Solve: $\log_{x^2+6x+8}(\underbrace{\log_{2x^2+2x+3}(x^2-2x)}_{t}) = 0$

$$\log_{x^2+6x+8} t = 0$$

$$t = (x^2+6x+8)^0$$

$$t = 1$$

$$\log_{2x^2+2x+3}(x^2-2x) = 1$$

$$x^2-2x = 2x^2+2x+3$$

$$x^2+4x+3=0$$

$$x = -1, \cancel{-3}$$

@ $x=-1$

$$\log_3(\log_3 3) = 0$$

@ $x=-3$ (rejected)

$\because$ x^2+6x+8 becomes -1

which is N.P

QUESTION [JEE Advanced 2011]

Let (x_0, y_0) be the solution of the following equations

$(2x)^{\ln 2} = (3y)^{\ln 3}$

$3^{\ln x} = 2^{\ln y}$

Then x_0 is

(A) 1/6

(B) 1/3

(C) 1/2

(D) 6

$$(2x)^{\ln 2} = (3y)^{\ln 3}$$

$$\ln (2x)^{\ln 2} = \ln \left((3y)^{\ln 3}\right)$$

$$\ln 2 \cdot \ln 2x = \ln 3 \cdot \ln 3y$$

$$\ln 2 \cdot (\ln 2 + \ln x) = \ln 3 \cdot (\ln 3 + \ln y)$$

$$(\ln 2)^2 + \ln 2 \cdot \ln x = (\ln 3)^2 + \ln 3 \cdot \frac{\ln x \cdot \ln 3}{\ln 2}$$

$$(\ln 2)^2 - (\ln 3)^2 = \frac{\ln x \cdot (\ln 3)^2}{\ln 2} - \ln 2 \cdot \ln x$$

$$\cancel{(\ln 2)^2 - (\ln 3)^2} = \ln x \left(\frac{(\ln 3)^2}{\ln 2} - \ln 2\right) = \ln x \, \frac{\cancel{\left((\ln 3)^2 - (\ln 2)^2\right)}}{\ln 2}$$

$$3^{\ln x} = 2^{\ln y}$$

$$\ln (3^{\ln x}) = \ln (2^{\ln y})$$

$$\ln x \cdot \ln 3 = \ln y \cdot \ln 2$$

$$\ln y = \frac{\ln x \cdot \ln 3}{\ln 2}$$

$$-1 = \frac{\ln x}{\ln 2}$$

$$\ln x = -\ln 2$$

$$\ln x = \ln 2^{-1}$$

$$x = 2^{-1}$$

$$x = 1/2.$$

QUESTION [JEE Advanced 2012]

The values of $6+\log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\ldots\ldots}}}\right)$ is

let E

let $x=\sqrt{4-\frac{1}{3\sqrt{2}}\underbrace{\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}---}}}}$

$x=\sqrt{4-\frac{1}{3\sqrt{2}}\cdot x}$

$x^2=4-\frac{x}{3\sqrt{2}}$

$3\sqrt{2}x^2=12\sqrt{2}-x$

$3\sqrt{2}x^2+x-12\sqrt{2}=0$

$3\sqrt{2}x^2+9x-8x-12\sqrt{2}=0$

$3x(\sqrt{2}x+3)-4\sqrt{2}(\sqrt{2}x+3)=0$

$(3x-4\sqrt{2})(\sqrt{2}x+3)=0$

$x=\frac{4\sqrt{2}}{3},\ \cancel{\frac{-3}{\sqrt{2}}}$

$x = \frac{4\sqrt{2}}{3}$

Now from given Quest

$$E = 6 + \log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}} \cdot x\right)$$

$$= 6 + \log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}} \cdot \frac{4\sqrt{2}}{3}\right)$$

$$= 6 + \log_{\frac{3}{2}}\left(\frac{4}{9}\right)$$

$$= 6 + \log_{\frac{3}{2}}(2/3)^2$$

$$= 6 + 2\log_{3/2}(2/3)$$

$$= 6 - 2 = 4$$

QUESTION

If $\log_{12} 27 = a$, then $\log_6 16$ is equal to

(A) $4\left(\dfrac{3+a}{3-a}\right)$

(B) $4\left(\dfrac{3-a}{3+a}\right)$

(C) $2\left(\dfrac{3-a}{3+a}\right)$

(D) $2\left(\dfrac{3+a}{3-a}\right)$

$\log_{12} 27 = a$

$\log_{12} 3^3 = a$

$3\log_{12} 3 = a$

$3\dfrac{\log_2 3}{\log_2 12} = a$

$\dfrac{3\log_2 3}{\log_2 4 + \log_2 3} = a \Rightarrow \dfrac{3\log_2 3}{2+\log_2 3} = a \Rightarrow \dfrac{3t}{2+t} = a$

$3t = 2a + at$

$t = \dfrac{2a}{3-a}$

$\log_6 16 = \log_6 2^4 = 4\log_6 2 = \dfrac{4}{\log_2 6}$

$= \dfrac{4}{\log_2 2 + \log_2 3} = \dfrac{4}{1+\log_2 3}$

let $\log_2 3 = t$

$\log_6 16 = \dfrac{4}{1+t}$

$\log_6 16 = \dfrac{4}{1+\frac{2a}{3-a}}$

$= \dfrac{4(3-a)}{3+a}$

QUESTION

★★KCLS★★

Find the value of $\log_{54}(168)$ if $\log_7 12 = a$ and $\log_{12} 24 = b$.

$\log_7 12 \cdot \log_{12} 24 = ab$

$\log_7 24 = ab$

$$\log_{54}(168) = \frac{\log_7 168}{\log_7 54} = \frac{\log_7 7 + \log_7 24}{\log_7 3^3\cdot 2} = \frac{1 + \log_7 24}{3\log_7 3 + \log_7 2} = \frac{1+ab}{3\log_7 3 + \log_7 2}$$

$\log_7 12 = a \quad , \quad ab = \log_7 24$

$\log_7 3 + 2\log_7 2 = a \quad , \quad 3\log_7 2 + \log_7 3 = ab$

$\log_7 2 = ab - a$

$\log_7 3 + 2ab - 2a = a$

$\log_7 3 = 3a - 2ab$

$$\log_{54} 168 = \frac{1+ab}{9a - 6ab + ab - a} = \frac{1+ab}{8a - 5ab} = \frac{1+ab}{a(8-5b)}$$

$$\log_{a_1} a_2 \cdot \log_{a_2} a_3 \, \log_{a_3} a_4 \cdots \log_{a_{n-1}} a_n = \log_{a_1} a_n$$

Ex: $\log_7 12 \cdot \log_{12} 24 = \log_7 24$

QUESTION

Tah01

If $\log_{10} 5 = a$ and $\log_{10} 3 = b$, then :

(A) $\log_{30} 8 = \frac{3(1+a)}{b+1}$

(B) $\log_{30} 8 = \frac{3(1-a)}{b+1}$

(C) $\log_{243}(32) = \frac{(1-a)}{b}$

(D) $\log_{40}(15) = \frac{a+b}{3-2a}$

Logarithmic Equations Involving Modulus

- $\log_a x^{2n} = 2n \log_a |x|$
- $\log_a \sqrt{x^2} = \log_a |x|$
- $|x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$

QUESTION

$$\log_4(x^2-1)-\log_4(x-1)^2=\log_4\sqrt{(4-x)^2}$$

(A) $3+\sqrt{6}$

(B) $3-\sqrt{6}$

(C) 5

(D) −1

$$|a-b|=|b-a|$$
$$\| \quad |-(b-a)|=|-1|\,|b-a|$$
$$=|b-a|$$

$$\log_4(x^2-1)-\log_4(x-1)^2=\log_4|4-x|$$

$$\log_4\frac{x^2-1}{(x-1)^2}=\log_4|4-x|$$

$$\frac{\cancel{(x-1)}(x+1)}{(x-1)^{\cancel{2}}}=|4-x|$$

$$\frac{x+1}{x-1}=|x-4|\ ,\ x\neq 1$$

case① if $x-4\geq 0$ i.e $x\geq 4$

$$\frac{x+1}{x-1}=x-4\Rightarrow x+1=x^2-5x+4$$
$$x^2-6x+3=0$$
$$x=\frac{6\pm\sqrt{24}}{2}=3+\sqrt{6},\ \cancel{3-\sqrt{6}}$$

Case (ii) If $x-4<0 \Rightarrow x<4$

$$\frac{x+1}{x-1} = -(x-4)$$

$$x+1 = -(x^2-5x+4)$$

$$x^2-4x+5=0$$

$D<0$.

(No real roots)

QUESTION

$$2\log_8(2x)+\log_8(x^2+1-2x)=\frac{4}{3}$$

(A) -1

(B) 2

(C) -2

(D) 1

$2\log_8 2x+\log_8 (x-1)^2=\frac{4}{3}$

$2\log_8 2x+2\log_8 |x-1|=\frac{4}{3}$

$\log_8 2x+\log_8 |x-1|=2/3$

$\log_8(2x\cdot|x-1|)=2/3$

$2x\cdot|x-1|=8^{2/3}=(8^{1/3})^2=4$

$x\cdot|x-1|=2$

case (I) If $x-1\geq 0$ i.e $x\geq 1$

$x(x-1)=2$

$x^2-x-2=0$

$(x-2)(x+1)=0$

$x=2,-1$ rejected

Case (II) If $x-1<0\Rightarrow x<1$

$-x(x-1)=2$

$x^2-x+2=0$

(No real roots)

QUESTION

$$2\log_3(x-2)+\log_3(x-4)^2=0$$

Tahoz

$2\log_3(x-2)+2\log_3|x-4|=0$

(A) $3+\sqrt{2}$

(B) $3-\sqrt{2}$

(C) 3

(D) -3

QUESTION

$$|x-1|^{\log_3 x^2 - 2\log_x 9} = (x-1)^7$$

(A) 3

(B) 2

(C) 27

(D) 81

Clearly $x > 1$

$(x-1)^{\log_3 x^2 - 2\log_x 9} = (x-1)^7$

Case (I) $x - 1 = 1$

$x = 2$

Case (II) $\log_3 x^2 - 2\log_x 9 = 7$

$2\log_3 x - 2\cdot 2\cdot \log_x 3 = 7$

$2\log_3 x - \frac{4}{\log_3 x} = 7$

$2t - \frac{4}{t} = 7$

$2t^2 - 4 = 7t$

$2t^2 - 7t - 4 = 0$

$(\text{any +ve real})^{\text{any real}} \in R^+$

$|x-1| > 0$

$a^x = a^y$:
- $a = 1$
- $a = 0,\ x, y > 0$
- $a = -1,\ (-1)^x = (-1)^y$
- $x = y$ & both sides are defined

$$2t^2-8t+t-4=0$$

$$(2t+1)(t-4)=0$$

$$t=-\frac{1}{2},\ 4$$

$$\log_3 x=-\frac{1}{2},\ 4$$

$$x=3^{-1/2},\ 3^4$$

$$x=\frac{1}{\sqrt{3}},\ 81$$

QUESTION

$|x-1|^{\log_3 x^2 - 2\log_x 9} = (x-1)^7$

$(\text{any +ve real})^{\text{any real}} \in R^+$

(A) 3

(B) 2

(C) 27

(D) 81

Clearly $x>1$

$(x-1)^{\log_3 x^2 - 2\log_x 9} = (x-1)^7$

$\log_3\left((x-1)^{\left(\log_3 x^2 - 2\log_x 9\right)}\right) = \log_3 (x-1)^7$

$\left(\log_3 x^2 - 2\log_x 9\right)\cdot \log_3(x-1) = 7\log_3(x-1)$

$\left(2\log_3 x - 4\log_x 3\right)\cdot \log_3(x-1) - 7\log_3(x-1) = 0$

$\log_3(x-1)\left(2\log_3 x - \frac{4}{\log_3 x} - 7\right) = 0$

$\log_3(x-1) = 0$ or $2\log_3 x - \frac{4}{\log_3 x} - 7 = 0$

$x-1 = 3^0$ or $2t - \frac{4}{t} - 7 = 0$

$x = 2$

$2t^2 - 7t - 4 = 0$

$t = -1/2, 4$

$x = \cancel{\frac{1}{\sqrt{3}}}, 81$

QUESTION

Solve the following logarithmic equations:

1. $\log_3(x^2-3x-5)=\log_3(7-2x)$
2. $x^{0.5\log_{\sqrt{x}}(x^2-x)}=3^{\log_9 4}$
3. $25^{\log_{10}x}=5+4\times\log_{10}5$
4. $1+2\log_{x+2}5=\log_5(x+2)$
5. $2\log_2(\log_2 x)+\log_{\frac{1}{2}}(\log_2(2\sqrt{2}x))=1$

Tah03

$(x^2-x)^{0.5\log_{\sqrt{x}} x}=4^{\log_9 3}$

$(x^2-x)^{0.5\times 2}=4^{\frac{1}{2}}$

$x^2-x=2$

$x^2-x-2=0$

$(x-2)(x+1)=0$

$x=2, \cancel{-1}$

Saari Class Illustrations

Retry karni Hai

Today's BPP

QUESTION

Solve the following equations :

(i) $\log_{x-1} 3 = 2$

(ii) $\log_4\left(2\log_3(1 + \log_2(1 + 3\log_3 x))\right) = \frac{1}{2}$

(iii) $\log_3(1 + \log_3(2^x - 7)) = 1$

(iv) $\log_3(3^x - 8) = 2 - x$

(v) $\frac{\log_2(9-2^x)}{3-x} = 1$

QUESTION

Solve the following equations :

(vi) $\log_{5-x}(x^2 - 2x + 65) = 2$

(vii) $\log_{10} 5 + \log_{10}(x + 10) - 1 = \log_{10}(21x - 20) - \log_{10}(2x - 1)$

(viii) $x^{1+\log_{10} x} = 10x$

(ix) $2\left(\log_x \sqrt{5}\right)^2 - 3\log_x \sqrt{5} + 1 = 0$

(x) $3 + 2\log_{x+1} 3 = 2\log_3(x + 1)$

Answers:

i.	$\{1 + \sqrt{3}\}$	ii.	$\{3\}$	iii.	$\{4\}$	iv.	$\{2\}$
v.	$\{0\}$	vi.	$\{-5\}$	vii.	$\{3/2, 10\}$	viii.	$\{10^{-1}, 10\}$
ix.	$\{\sqrt{5}, 5\}$	x.	$\{-(3 - \sqrt{3})/3, 8\}$				

Solution to Previous TAH

QUESTION

Prove that $\frac{\log_2 24}{\log_{96} 2} - \frac{\log_2 192}{\log_{12} 2} = 3$

* TAH

TAH1:) Prove that $\dfrac{\log_2 24}{\log_{96} 2} - \dfrac{\log_2 192}{\log_{12} 2} = 3$

$\Rightarrow \log_2 24 \times \log_2 96 - \log_2 192 \times \log_2 12$

$\Rightarrow (\log_2 2^3 + \log_2 3)(\log_2 2^5 + \log_2 3) - (\log_2 2^6 + \log_2 3)(\log_2 2^2 + \log_2 3)$

$= (3 + \log_2 3)(5 + \log_2 3) - (6 + \log_2 3)(2 + \log_2 3)$

$= 15 + 3\log_2 3 + 5\log_2 3 + \cancel{\log_2 3 \cdot \log_2 3} - 12 - 6\log_2 3 - 2\log_2 3 - \cancel{\log_2 3 \cdot \log_2 3}$

$= 15 + \cancel{8\log_2 3} - 12 - \cancel{8\log_2 3}$

$= 15 - 12 = \underline{\underline{3}}$

$= RHS$

Hence Proved

Kriti Mathur

Raj.

$$\# \quad \frac{\log_2 24}{\log_{96} 2} - \frac{\log_2 192}{\log_{12} 2} = 3 \quad \text{(Tah-01)}$$

L.H.S.

$$\Rightarrow \log_2 24 \cdot \log_2 96 - \log_2 192 \cdot \log_2 12$$

$$\Rightarrow (\log_2 2 + \log_2 12) \cdot \log_2 96 \cancel{\log} - (\log_2 2 + \log_2 96) \log_2 12$$

$$\Rightarrow (1 + \log_2 12) \log_2 96 - (1 + \log_2 96) \log_2 12$$

$$\Rightarrow \log_2 96 + \cancel{\log_2 12 \cdot \log_2 96} - \log_2 12 - \cancel{\log_2 96 \cdot \log_2 12}$$

$$\Rightarrow \log_2 96 - \log_2 12$$

$$\Rightarrow \log_2 \left(\frac{96}{12}\right)$$

$$\Rightarrow \log_2 8$$

$$\Rightarrow \log_2 2^3$$

$$\Rightarrow \underline{3} \quad \underline{\text{proved}}.$$

Key concept.

$$\log_2 24 = \log_2 2 + \log_2 12$$

$$\log_2 192 = \log_2 2 + \log_2 96$$

Tah-01.

Prove that $\dfrac{\log_2 24}{\log_{96} 2} - \dfrac{\log_2 192}{\log_{12} 2} = 3$.

$\log_b a = \dfrac{1}{\log_a b}$ ☆

LHS.

$\Rightarrow \log_2 96 \cdot \log_2 24 - \log_2 12 \cdot \log_2 192$.

$\Rightarrow \log_2 (2^5 \times 3) \cdot \log_2 (2^3 \times 3) - \log_2 (2^2 \times 3) \cdot \log_2 (2^6 \times 3)$.

$\Rightarrow (\log_2 2^5 + \log_2 3) \cdot (\log_2 2^3 + \log_2 3) - (\log_2 2^2 + \log_2 3)(\log_2 2^6 + \log_2 3)$.

Let : $\log_2 3 = t$

$\Rightarrow (5+t)(3+t) - (2+t)(6+t)$

$\Rightarrow (15 + 5t + 3t + t^2) - (12 + 2t + 6t + t^2)$

$\Rightarrow \cancel{t^2} + \cancel{8t} + 15 - \cancel{t^2} - \cancel{8t} - 12$

$\Rightarrow 3 = \underline{\text{RHS}}$ (Proved).

krish keshri
jharkhand

QUESTION

Simplify & compute :

$$\frac{\log_5 250}{\log_{50} 5} - \frac{\log_5 10}{\log_{1250} 5}$$

Tah-02. Simplify & compute :

$$\Rightarrow \frac{\log_5 250}{\log_{50} 5} - \frac{\log_5 10}{\log_{1250} 5}.$$

$$\Rightarrow \log_5 50 \cdot \log_5 250 - \log_5 1250 \cdot \log_5 10.$$

$$\Rightarrow \log_5 (5^2 \times 2) \cdot \log_5 (5^3 \times 2) - \log_5 (5^4 \times 2) \cdot \log_5 (5 \times 2)$$

$$\Rightarrow (\log_5 5^2 + \log_5 2) \cdot (\log_5 5^3 + \log_5 2) - (\log_5 5^4 + \log_5 2) \cdot (\log_5 5 + \log_5 2).$$

Let : $\log_5 2 = t$

$$\Rightarrow (2+t)(3+t) - (4+t)(1+t).$$

$$\Rightarrow (6 + 2t + 3t + t^2) - (4 + 4t + t + t^2).$$

$$\Rightarrow \cancel{t^2} + \cancel{5t} + 6 - \cancel{t^2} - \cancel{5t} - 4$$

$\Rightarrow$ 2 Ans.

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PW

$$\frac{\log_5 250}{\log_{50} 5} - \frac{\log_5 10}{\log_{1250} 5}$$ Tah - 02

$\Rightarrow \log_5 250 \cdot \log_5 50 - \log_5 10 \cdot \log_5 1250$

$\Rightarrow \log_5 250 \cdot (\log_5 5 + \log_5 10) - \log_5 10 \cdot (\log_5 5 + \log_5 250)$

$\Rightarrow \log_5 250 \cdot (1 + \log_5 10) - \log_5 10 (1 + \log_5 250)$

$\Rightarrow \log_5 250 + \cancel{\log_5 250 \cdot \log_5 10} - \log_5 10 - \cancel{\log_5 10 \cdot \log_5 250}$

$\Rightarrow \log_5 250 - \log_5 10$

$\Rightarrow \log_5 \left(\frac{250}{10}\right)$

$\Rightarrow \log_5 25$

$\Rightarrow \log_5 5^2$

$\Rightarrow \underline{2}$ Ans

QUESTION [JEE Advanced 2018]

The value of $\left((\log_2 9)^2\right)^{\frac{1}{\log_2(\log_2 9)}} \times \left(\sqrt{7}\right)^{\frac{1}{\log_4 7}}$ is

Tah-03.

The value of $\left((\log_2 9)^2\right)^{\frac{1}{\log_2(\log_2 9)}} \times (\sqrt{7})^{\frac{1}{\log_4 7}}$ is:

Let: $\log_2 9 = t$

$$\log_a b = \frac{1}{\log_b a}$$

$$\Rightarrow (x^2)^{\frac{1}{\log_2 x}} \times (7)^{\frac{1}{2}\log_7 4}$$

$$(a)^{\log_a x} = x$$

$$\Rightarrow (x)^{2\log_x 2} \times (7)^{\frac{1}{2}\log_7 4}.$$

$$\Rightarrow x^{\log_x (2)^2} \times 7^{\log_7 (4)^{1/2}}.$$

$$\Rightarrow 2^2 \times 2 = 2^3 = 8$$ Ans.

The value of $\left((\log_2 9)^2\right)^{\frac{1}{\log_2(\log_2 9)}} \times (\sqrt{7})^{\frac{1}{\log_4 7}}$ is

Tah 03

$$\left((\log_2 9)^2\right)^{\frac{1}{\log_2 \log_2 9}} \times (\sqrt{7})^{\frac{1}{\log_4 7}}$$

$$(\log_2 9)^{2\log_{\log_2 9} 2} \times 7^{\frac{1}{2}\log_7 4}$$

$$(\log_2 9)^{\log_{\log_2 9} 4} \times 7^{\log_7 (4)^{1/2}}$$

$$4^{\log_{\log_2 9} \log_2 9} \times (4)^{1/2}$$

$$4 \times 4^{1/2} = 8$$

QUESTION

Indicate all correct alternatives, where base of the log is 2.

The equation $x^{\frac{3}{4}(\log_2 x)^2+\log_2 x-\frac{5}{4}}=\sqrt{2}$ has :

(A) At least one real solution

(B) Exactly 3 real solutions

(C) Exactly one irrational solution

(D) Imaginary roots

Tah-04 : Indicate all correct alternatives, where base of the log is 2. The equation :

$$\Rightarrow x^{\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4}} = \sqrt{2} \text{ has :}$$

A) At least one real solⁿ.

B) Exactly 3 real solⁿ.

C) Exactly one irrational solⁿ.

D) Imaginary roots.

$\Rightarrow$ Take log base 2 on both sides.

$$\Rightarrow \log_2 x \left[\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4}\right] = \frac{1}{2}\log_2 2.$$

$\Rightarrow$ let $\log_2 x = t$ *

$$\Rightarrow t\left[\frac{3}{4}t^2 + t - \frac{5}{4}\right] = \frac{1}{2}$$

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$$\Rightarrow \frac{3}{4}t^3 + t^2 - \frac{5}{4}t - \frac{1}{2} = 0.$$

$$\Rightarrow \frac{3t^3 + 4t^2 - 5t - 2}{4} = 0.$$

$$\Rightarrow 3t^3 + 4t^2 - 5t - 2 = 0.$$

put, $t = 1$

$= 3 + 4 - 5 - 2 = 0$

$(t-1)$ is a factor.

$$\Rightarrow 3t^2(t-1) + 7t(t-1) + 2(t-1) = 0.$$

$$\Rightarrow (t-1)(3t^2 + 7t + 2) = 0$$

$$\Rightarrow (t-1)(3t+1)(t+2) = 0$$

$$\Rightarrow \boxed{t = 1, -\frac{1}{3}, -2.}$$

1) $\log_2 x = 1 \Rightarrow x = 2^1.$

2) $\log_2 x = -\frac{1}{3} \Rightarrow x = 2^{-1/3}.$ $= \frac{1}{\sqrt[3]{2}}.$

3) $\log_2 x = -2$

$\Rightarrow x = 2^{-2} = \frac{1}{4}.$

TAH 4.) Indicate all correct alternatives, where base of the log is 2.

The equation $x^{\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4}} = \sqrt{2}$ has:

A.) At least one real solution

B.) Exactly 3 real solutions

C.) Exactly one irrational solution

D.) Imaginary roots

$$x^{\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4}} = \sqrt{2}$$

take log both sides with base 2

$$\left[\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4}\right] \cdot \log_2 x = \log_2 \sqrt{2}$$

$$\left[\frac{3}{4}(\log_2 x)^2 + \log_2 x - \frac{5}{4}\right] \cdot \log_2 x = \frac{1}{2}$$

Let $\log_2 x = t$

$$\left[\frac{3}{4}t^2 + t - \frac{5}{4}\right] t = \frac{1}{2}$$

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$$= (3t^2 + 4t - 5)t = 2$$

$$P = 3t^3 + 4t^2 - 5t - 2 = 0$$

at $t = 1$

$P = 0 \because (t-1)$ is a factor

$$3t^2(t-1) + 7t(t-1) + 2(t-1) = 0$$

$$(t-1)(3t^2 + 7t + 2) = 0$$

$$(t-1)(3t+1)(t+2) = 0$$

$t = 1$, $t = -\frac{1}{3}$, $t = -2$

$\log_2 x = 1$, $\log_2 x = -\frac{1}{3}$, $\log_2 x = -2$

$x = 2$, $x = \frac{1}{\sqrt[3]{2}}$, $x = \frac{1}{4}$

(A), (B) and (C) - Ans.

QUESTION

The equation $x^{\left[(\log_3 x)^2-\frac{9}{2}\log_3 x+5\right]}=3\sqrt{3}$ has

(A) Exactly 3 real solutions

(B) At least one real solution

(C) Exactly one irrational solution

(D) Complex roots

Tah-05. The equation $x^{\left[(\log_3 x)^2 - \frac{9}{2}\log_3 x + 5\right]} = 3\sqrt{3}$ has:

A) Exactly 3 real soln.

B) At least one real soln.

C) Exactly one irration soln.

D) Complex roots.

$\Rightarrow$ Taking log base 3 on both sides.

$\Rightarrow \log_3 x \left[(\log_3 x)^2 - \frac{9}{2}\log_3 x + 5\right] = \frac{3}{2}\log_3 3.$

$\Rightarrow$ let: $\log_3 x = t$

$\Rightarrow t\left[t^2 - \frac{9}{2}t + 5\right] = \frac{3}{2}$

$\Rightarrow t^3 - \frac{9}{2}t^2 + 5t - \frac{3}{2} = 0$

$\Rightarrow \frac{2t^3 - 9t^2 + 10t - 3}{2} = 0$

$\Rightarrow 2t^3 - 9t^2 + 10t - 3 = 0$

Put, $t = 1$

$\Rightarrow 2 - 9 + 10 - 3 = 0.$

$(t-1)$ is a factor.

$\Rightarrow 2t^2(t-1) - 7t(t-1) + 3(t-1) = 0$

$\Rightarrow (t-1)(2t^2 - 7t + 3) = 0$

$\Rightarrow (t-1)(2t-1)(t-3) = 0$

$\Rightarrow \boxed{t = 1, \frac{1}{2}, 3}.$

1) $\log_3 x = 1 \Rightarrow x = 3.$

2) $\log_3 x = \frac{1}{2} \Rightarrow x = 3^{1/2} = \sqrt{3}.$

3) $\log_3 x = 3 \Rightarrow x = 3^3 = 27.$

TAHS·) The equation $x^{\left[(\log_3 x)^2 - \frac{9}{2}\log_3 x + 5\right]} = 3\sqrt{3}$

taking log both sides with base 3

$$\left[(\log_3 x)^2 - \frac{9}{2}\log_3 x + 5\right]\cdot \log_3 x = \log_3 3\sqrt{3}$$

$$\left[(\log_3 x)^2 - \frac{9}{2}\log_3 x + 5\right]\cdot \log_3 x = \frac{3}{2}$$

let $\log_3 x = t$

$$\left[t^2 - \frac{9}{2}t + 5\right]t = \frac{3}{2}$$

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$$[2t^2 - 9t + 10]t = 3$$

$$2t^3 - 9t^2 + 10t - 3 = 0$$

$$2t^2(t-1) - 7t(t-1) + 3(t-1) = 0$$

$$(t-1)(2t^2 - 7t + 3) = 0$$

$$(t-1)(2t^2 - 6t - t + 3) = 0$$

$$(t-1)[2t(t-3) - 1(t-3)]$$

$$(t-1)(2t-1)(t-3) = 0$$

$t = 1$, $t = \frac{1}{2}$, $t = 3$

$\log_3 x = 1$, $\log_3 x = \frac{1}{2}$, $\log_3 x = 3$

$x = 3$, $x = \sqrt{3}$, $x = 27$

(A) Exactly 3 real solutions

(B) At least one real solution

(C) Exactly one irrational solution

— Ans.

Solution to Previous KTKs

QUESTION

(KTK 1)

Solve the following inequalities:

(1) $\frac{x}{x-5} > \frac{1}{2}$

(2) $\frac{x-5}{x-9} \leq 0$

(3) $x \leq 3 - \frac{1}{x-1}$

(4) $\frac{1}{x} \leq 1$

(5) $\frac{5x}{3x-1} \leq 0$

Ans. (1) $x \in (-\infty, -5) \cup (5, \infty)$, (2) $x \in (-\infty, 0) \cup [1, \infty)$, (3) $x \in [5, 9)$, (4) $x \in \left[0, \frac{1}{3}\right)$, (5) $x \in (-\infty, 1) \cup \{2\}$

KTK-1.

Solve the following inequalities :

(1) $\frac{x}{x-5} > \frac{1}{2}$.

$\Rightarrow \frac{x}{x-5} - \frac{1}{2} > 0$

$\Rightarrow \frac{2x - x + 5}{2(x-5)} > 0$

$\Rightarrow \frac{x-5}{2x-10} > 0$

$\Rightarrow$ + -5 − 5 +

$\Rightarrow \boxed{x \in (-\infty, -5) \cup (5, \infty).}$

(2) $\frac{x-5}{x-9} \leq 0$.

$\Rightarrow$ + 5 − 9 +

$\Rightarrow \boxed{x \in [5, 9).}$

(3) $x \leq 3 - \frac{1}{x-1}$.

$\Rightarrow 3 - \frac{1}{x-1} - x \geq 0$

$\Rightarrow \frac{3x - 3 - 1 - x^2 + x}{(x-1)} \geq 0$

$\Rightarrow \frac{-x^2 + 4x - 4}{(x-1)} \geq 0$

$\Rightarrow \frac{x^2 - 4x + 4}{(x-1)} \leq 0$

$\Rightarrow \frac{(x-2)^2}{(x-1)} \leq 0$; $\boxed{x = 2}$ is possible.

$\Rightarrow$ − 1 +

$\Rightarrow \boxed{x \in (-\infty, 1) \cup \{2\}.}$

(4) $\frac{1}{x} \leq 1$.

$\Rightarrow \frac{1}{x} - 1 \leq 0$.

$\Rightarrow \frac{1-x}{x} \leq 0$.

$\Rightarrow \frac{x-1}{x} \geq 0$.

$\Rightarrow$ + 0 − 1 +

$\Rightarrow \boxed{x \in (-\infty, 0) \cup [1, \infty).}$

(5) $\frac{5x}{3x-1} \leq 0$.

$\Rightarrow$ + 0 − $\frac{1}{3}$ +

$\Rightarrow \boxed{x \in \left[0, \frac{1}{3}\right).}$

—x—

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KTK - 01

1. $\frac{x}{x-5} > \frac{1}{2}$

$\frac{x}{x-5} - \frac{1}{2} > 0$

$\frac{2x-(x-5)}{2(x-5)} > 0$

$\frac{x+5}{2x-10} > 0 \Rightarrow x \in (-\infty, -5) \cup (5, \infty)$

2. $\frac{x-5}{x-9} \leq 0$

$x \in [5, 9)$

iii) $x \leq 3 - \frac{1}{x-1}$

$$x + \frac{1}{x-1} - 3 \leq 0$$

$$x - 3 + \frac{1}{x-1} \leq 0$$

$$\frac{(x-3)(x-1)}{x-1} \leq 0$$

$$\frac{x^2 - 4x + 4}{x-1} \leq 0$$

$$\frac{(x-2)^2}{x-1} \leq 0$$

$$x \in (-\infty, 1) \cup \{2\}$$

iv) $\frac{1}{x} \leq 1$

$$\frac{x-1}{x} \geq 0$$

$$x \in (-\infty, 0) \cup [1, \infty)$$

v. $\frac{5x}{3x-1} \leq 0$

$$x \in [0, \tfrac{1}{3})$$

QUESTION

(KTK 2)

Complete solution set of inequality $\frac{(x+2)(x+3)}{(x-2)(x-3)} \leq 1$ is

(A) $(-\infty, 0]$

(B) $(-\infty, 0] \cup (2, 3)$

(C) $[2, 3]$

(D) $(-\infty, 2) \cup (3, \infty)$

Ans. B

KTK-2.

Q. Complete solution set of inequality $\dfrac{(x+2)(x+3)}{(x-2)(x-3)} \leq 1.$

$\Rightarrow \quad \dfrac{(x+2)(x+3)}{(x-2)(x-3)} - 1 \leq 0.$

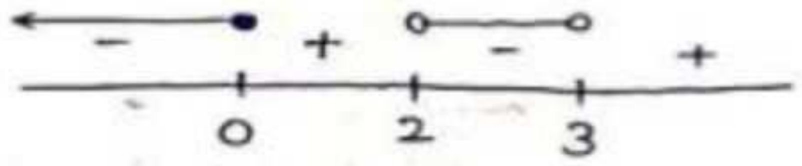
$\Rightarrow \quad \dfrac{(x+2)(x+3) - (x-2)(x-3)}{(x-2)(x-3)} \leq 0.$

$\Rightarrow \quad \dfrac{[x^2+5x+6] - [x^2-5x+6]}{(x-2)(x-3)} \leq 0.$

$\Rightarrow \quad \dfrac{\cancel{x^2}+5x+\cancel{6}-\cancel{x^2}+5x-\cancel{6}}{(x-2)(x-3)} \leq 0$

$\Rightarrow \quad \dfrac{10x}{(x-2)(x-3)} \leq 0.$

$\Rightarrow$



$\Rightarrow \quad \boxed{x \in (-\infty, 0] \cup (2, 3).}$

Q complete soln set of inequality

$$\frac{(x+2)(x+3)}{(x-2)(x-3)} \le 1 \text{ is}$$

KTK-02

(A) $(-\infty, 0]$

(B) $(-\infty, 0] \cup (2,3)$

(C) $[2,3]$

(d) $(-\infty, 2) \cup (3, \infty)$

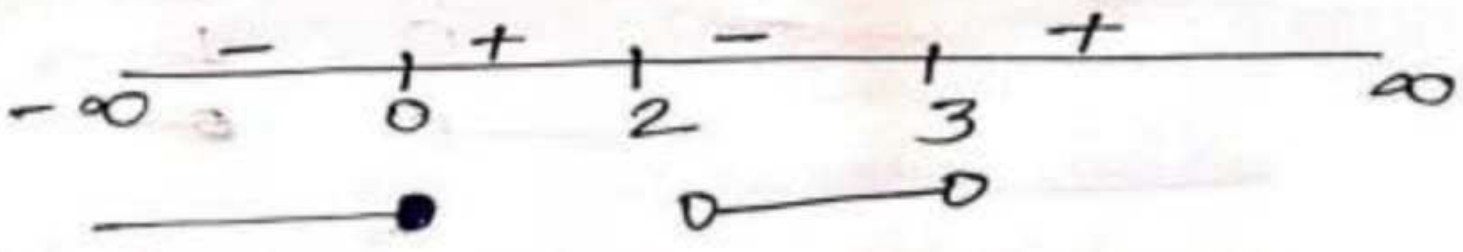
Soln

$$\Rightarrow \frac{(x+2)(x+3)}{(x-2)(x-3)} - 1 \le 0$$

$$\Rightarrow \frac{(x+2)(x+3) - (x-2)(x-3)}{(x-2)(x-3)} \le 0$$

$$\Rightarrow \frac{\cancel{x^2} + 5x + \cancel{6} - (\cancel{x^2} - 5x + \cancel{6})}{(x-2)(x-3)} \le 0$$

$$\Rightarrow \frac{10x}{(x-2)(x-3)} \le 0$$

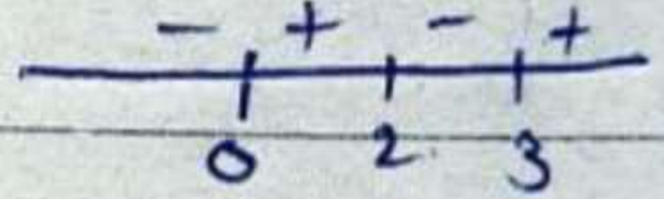


$x \in (-\infty, 0] \cup (2, 3)$ Ans

KTK 2

$$\frac{(x+2)(x+3)}{(x-2)(x-3)} \leq 1$$

$$\frac{(x+2)(x+3)-(x-2)(x-3)}{(x-2)(x-3)} \leq 0$$



$$\frac{\cancel{x^2}+5x+\cancel{6}-\cancel{x^2}+5x-\cancel{6}}{(x-2)(x-3)} \leq 0$$

$x \in (-\infty, 0] \cup (2,3)$

(B)

$$\frac{2(5x)}{(x-2)(x-3)} \leq 0$$

QUESTION

(KTK 3)

Find sum of all integral values of x satisfying $\frac{x^2-5x+6}{x^2-x-12} \leq 0$.

Ans. 3

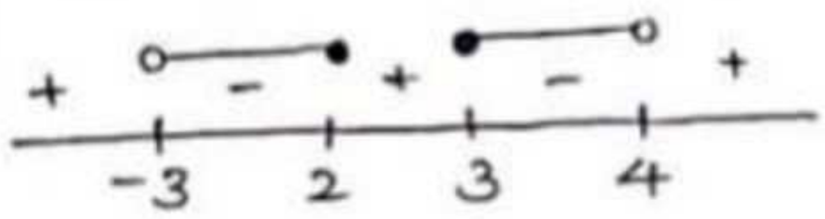
KTK-3

Q. Find the sum of all integral values of x satisfying

$$\frac{x^2-5x+6}{x^2-x-12} \le 0.$$

$$\Rightarrow \frac{(x-2)(x-3)}{(x-4)(x+3)} \le 0$$

$\Rightarrow$



$$\Rightarrow x \in (-3, 2] \cup [3, 4).$$

Sum of Integral value :

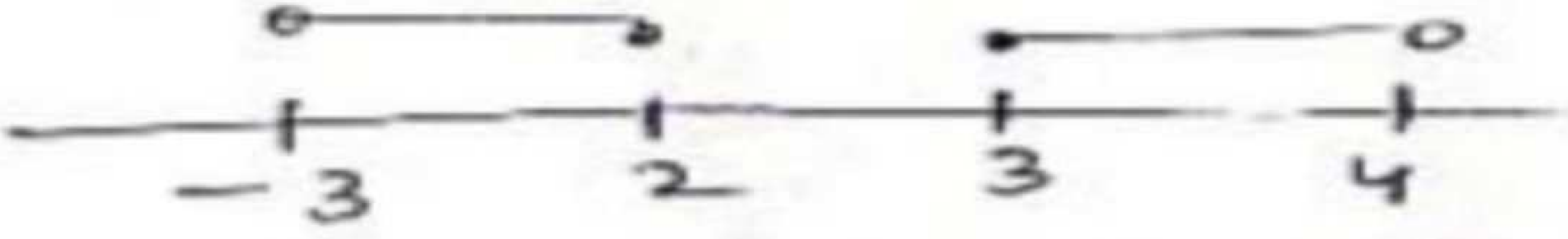
$$\Rightarrow \cancel{(-2)} + \cancel{(-1)} + 0 + \cancel{1} + \cancel{2} + 3$$

$\Rightarrow$ (3) Ans.

KTK - 03

$$\frac{x^2-5x+6}{x^2-x-12} \le 0$$

$$\frac{(x-3)(x-2)}{(x-4)(x+3)} \le 0$$



$(-3, 2] \cup [3, 4)$

QUESTION

(KTK 4)

Which of the following does not hold true for the expression

$E = \sqrt{x^2 - 2x + 1} - \sqrt{x^2 + 2x + 1}$?

(A) $E = 2$ if $x \leq -1$

(B) $E = -2x$ if $-1 < x < 1$

(C) $E = -2$ if $x \geq 1$

(D) $E = -2$ for all x

Ans. D

KTK - 04.

Q. Which of the following does not hold true for the expression $E = \sqrt{x^2 - 2x + 1} - \sqrt{x^2 + 2x + 1}$?

A) $E = 2$ if $x \le -1$. $\Rightarrow E = \sqrt{(x-1)^2} - \sqrt{(x+1)^2}$

B) $E = -2x$ if $-1 < x < 1$. $\Rightarrow \boxed{E = |x-1| - |x+1|}$ ★

C) $E = -2$ if $x \ge 1$.

D) $E = -2$ for all x.

Option check :

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(A) $x \le -1$

$E = -(x-1) + (x+1)$

$= \cancel{-x} + 1 + \cancel{x} + 1$

$= 2$ ✓

(B) $-1 < x < 1$

$E = -(x-1) - (x+1)$

$= -x \cancel{+1} - x \cancel{-1}$

$= -2x$ ✓

(C) $x \ge 1$

$E = (x-1) - (x+1)$

$= \cancel{x} - 1 \cancel{-x} - 1$

$= -2$ ✓

(D) For all x :

$\boxed{E = -2}$ for all x.

ये गलत है !

Ex : Option A & B.

Q. Which of the following does not hold true for the expression.

$$E = \sqrt{x^2 - 2x + 1} - \sqrt{x^2 + 2x + 1} = ?$$

(A) $E = 2$ if $x \le -1$

(B) $E = -2x$ if $-1 < x < 1$

(C) $E = -2$ if $x \ge 1$

(D) $E = -2$ for all x.

KTK-04

Soln

$$E = \sqrt{x^2 - 2x + 1} - \sqrt{x^2 + 2x + 1}$$
$$= \sqrt{x^2 - x - x + 1} - \sqrt{x^2 + x + x + 1}$$
$$= \sqrt{(x-1)(x-1)} - \sqrt{(x+1)(x+1)}$$
$$= \sqrt{(x-1)^2} - \sqrt{(x+1)^2}$$
$$= |x-1| - |x+1|$$

$x \ge 1$	$-1 < x < 1$	$x \le -1$
$\Rightarrow x - 1 - x - 1$	$\Rightarrow -x + 1 - x - 1$	$\Rightarrow -(x-1) - \{-(x+1)\}$
$\Rightarrow -2$	$\Rightarrow \underline{-2x}$	$\Rightarrow -x + 1 + x + 1$
		$\Rightarrow \underline{2}$.

Ans $\Rightarrow$ (A), (B) & (C)

QUESTION

KTK 05

If $x \in [-5, 7]$, then number of integral values of x satisfying $\frac{2x+3}{x^2+x-12} < \frac{1}{2}$ is

(A) 5

(B) 6

(C) 7

(D) 8

Ans. C

Q. If $x \in [-5, 7]$, then number of integral value of x satisfying $\dfrac{2x+3}{x^2+x-12} < \dfrac{1}{2}$ is :

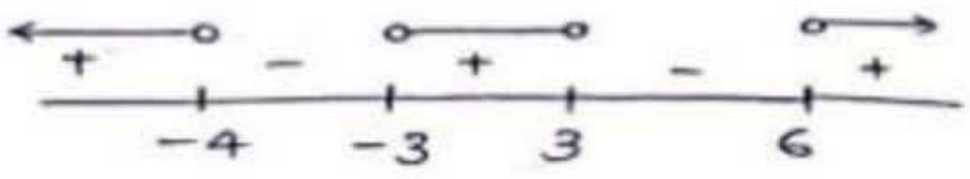
$$\Rightarrow \frac{2x+3}{x^2+x-12} - \frac{1}{2} < 0$$

$$\Rightarrow \frac{4x+6-x^2-x+12}{2(x^2+x+12)} < 0$$

$$\Rightarrow \frac{-x^2+3x+18}{x^2+x-12} < 0$$

$$\Rightarrow \frac{x^2-3x-18}{x^2+x-12} > 0$$

$$\Rightarrow \frac{(x-6)(x+3)}{(x+4)(x-3)} > 0 \Rightarrow$$



$$\Rightarrow x \in (-\infty, -4) \cup (-3, 3) \cup (6, \infty).$$

$$\Rightarrow \overset{\checkmark}{-5}, \overset{\times}{-4}, \overset{\times}{-3}, \overset{\checkmark}{-2}, \overset{\checkmark}{-1}, \overset{\checkmark}{0}, \overset{\checkmark}{1}, \overset{\checkmark}{2}, \overset{\times}{3}, \overset{\times}{4}, \overset{\times}{5}, \overset{\times}{6}, \overset{\checkmark}{7}.$$

$\Rightarrow$ 7 Ans.

Q) if $x \in [-5, 7]$, then number of integral values of x satisfying —

$$\frac{2x+3}{x^2+x-12} < \frac{1}{2} \text{ is}$$

KTK-07

Sol.

(A) 5

(B) 6

(C) 7

(D) 8

$$\Rightarrow \frac{2x+3}{x^2+x-12} < \frac{1}{2}$$

$$\Rightarrow \frac{2x+3}{x^2+x-12} - \frac{1}{2} < 0$$

$$\Rightarrow \frac{4x+6-x^2-x+12}{(x^2+x-12)2} < 0$$

$$\Rightarrow \frac{-x^2+3x+18}{x^2+x-12} < 0$$

$$\Rightarrow \frac{x^2-3x-18}{x^2+x-12} > 0$$

$$\Rightarrow \frac{x^2-6x+3x-18}{x^2+4x-3x-12} > 0$$

$$\Rightarrow \frac{(x-6)(x+3)}{(x+4)(x-3)} > 0$$

+   −   +   −   +

$-\infty$ -4 -3 3 6 ∞

$x \in (-\infty, -4)\ (-3, 3)\ (6, \infty)$ — given $x \in [-5, 7]$

finaly $x = \{-5, -2, -1, 0, 1, 2, 7\}$ No = 7 Ans

QUESTION

KTK 06

Solution set of the inequality $x + 1 \leq \frac{6}{x}$ is

(A) $(0, 2]$

(B) $[-3, 2)$

(C) $(-\infty, -3] \cup (0, 2]$

(D) $[-3, 0) \cup (2, \infty)$

Ans. C

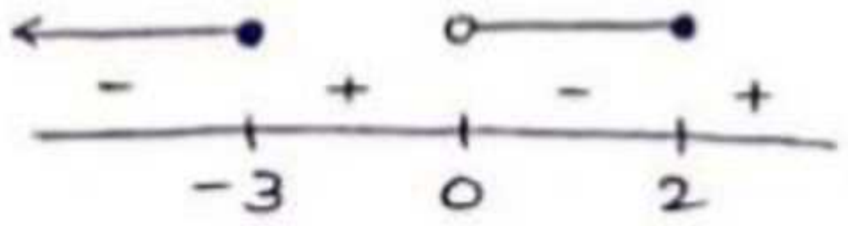
KTK-08.

Q. Solution set of the inequality $x+1 \leq \frac{6}{x}$ is :

$$\Rightarrow x+1-\frac{6}{x} \leq 0.$$

$$\Rightarrow \frac{x^2+x-6}{x} \leq 0.$$

$$\Rightarrow \frac{(x+3)(x-2)}{x} \leq 0.$$

$\Rightarrow$ 

$$\Rightarrow \boxed{x \in (-\infty,-3] \cup (0,2]}$$ Ans.

QUESTION

KTK 07

The set of all values of x for which $\frac{(x+1)(x-3)^2(x-5)(x-4)^3(x-2)}{x} < 0$ is

(A) $(-\infty, -1) \cup (0, 2) \cup (4, 5)$

(B) $(-1, 0) \cup (2, 4) \cup (5, \infty)$

(C) $(-1, 0) \cup (2, 3) \cup (4, 5)$

(D) $(-\infty, -1) \cup (0, 2) \cup [3, 5)$

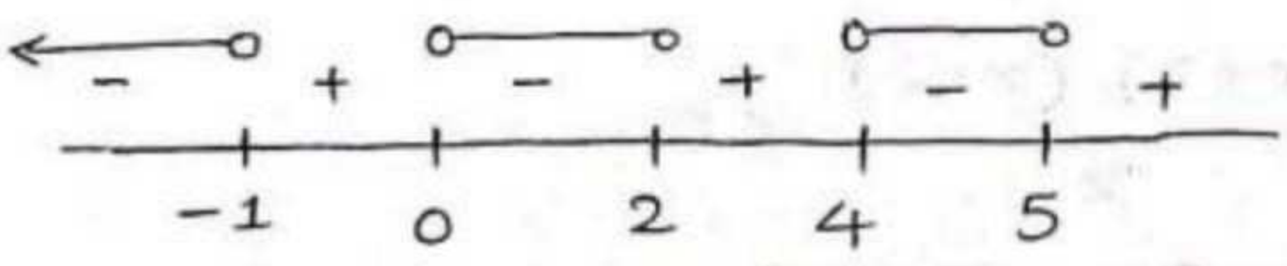
Ans. A

KTK-09.

Q. The set of all values of x for which

$$\Rightarrow \frac{(x+1)(x-3)^2(x-5)(x-4)^3(x-2)}{x} < 0 \text{ is :}$$

($x=3$ is not possible.)

$\Rightarrow$ 

$\therefore$

$x=3$ हटाने की जरूरत ही नहीं है बिचारा वो आ ही नहीं रहा है!

$$\Rightarrow \boxed{x \in (-\infty,-1) \cup (0,2) \cup (4,5).} \text{ Ans.}$$

—x—

QUESTION

KTK 8

Which of the following sets does not satisfy the inequality $\frac{1}{x-2}+\frac{1}{x-1}\geq\frac{1}{x}$?

A $(-\sqrt{2},0)$

B $(1,\sqrt{2})$

C $(2,\infty)$

D $(0,1)$

Ans. D

KTK-10

Q. Which of the following sets does not satisfy the inequality $\frac{1}{x-2}+\frac{1}{x-1}\geq\frac{1}{x}$?

A) $(-\sqrt{2},0)$

B) $(1,\sqrt{2})$

C) $(2,\infty)$

D) $(0,1)$.

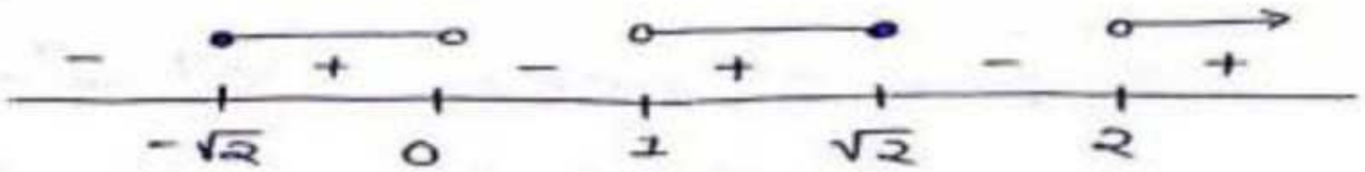
$$\Rightarrow \frac{1}{x-2}+\frac{1}{x-1}-\frac{1}{x}\geqslant 0.$$

$$\Rightarrow \frac{x(x-1)+x(x-2)-(x-1)(x-2)}{x(x-1)(x-2)}\geqslant 0.$$

$$\Rightarrow \frac{x^2-x+x^2-2x-(x^2-2x-x+2)}{x(x-1)(x-2)}\geqslant 0.$$

$$\Rightarrow \frac{\cancel{x^2}-\cancel{x}+x^2-\cancel{2x}-\cancel{x^2}+\cancel{2x}+\cancel{x}-2}{x(x-1)(x-2)}\geqslant 0.$$

$$\Rightarrow \frac{x^2-2}{x(x-1)(x-2)}\geqslant 0.$$

$\Rightarrow$ 

$$\Rightarrow \boxed{x\in[-\sqrt{2},0)\cup(1,\sqrt{2}]\cup(2,\infty)}.$$

$\because$ $(0,1)$ does not satisfy the inequality.

—x—

Home Challenge-03

Positive integers a and b satisfy the condition $\log_2[\log_{2^a}(\log_{2^b}(2^{1000}))] = 0$. Then the possible values of $a + b$ is/are:

(A) 501

(B) 252

(C) 128

(D) 66

HOME - CHALLENGE - 03

Q. Positive integers a and b satisfy the condition $\log_2[\log_2 a(\log_2 b(2^{1000}))] = 0$. Then the possible values of $a+b$ is/are.

A) 501.
B) 252.
C) 128.
D) 66.

$\Rightarrow \log_2[\log_2 a(\log_2 b(2^{1000}))] = 0$.

$\Rightarrow \log_2\left[\frac{1}{a}\log_2\left(\frac{1000}{b}\underbrace{\log_2 2}_{1}\right)\right] = 0$.

$\Rightarrow \log_2\left[\frac{1}{a}\log_2\left(\frac{1000}{b}\right)\right] = 0$

$\Rightarrow \frac{1}{a}\log_2\left(\frac{1000}{b}\right) = 1$.

$\Rightarrow \frac{1000}{b} = 2^a \Rightarrow \boxed{2^a \times b = 1000}$

(1) $2^a \times b = 2^1 \times 500$
$\Rightarrow a = 1,\ b = 500$
$\Rightarrow a+b = 501$.

(2) $2^a \times b = 2^2 \times 250$
$\Rightarrow a = 2,\ b = 250$
$\Rightarrow a+b = 252$.

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(3) $2^a \times b = 2^3 \times 125$
$\Rightarrow a = 3,\ b = 125$
$\Rightarrow a+b = 128$

—x—

THANK YOU